UNIT 6 INTRODUCTION TO PROBABILITY

Unit outcomes

After Completing this unit, you should be able to:

- understand the concept of certain, uncertain and impossible outcomes.
- know specific facts about event, sample space and probability of simple events.

Introduction

When you buy a lottery ticket you cannot be 100% sure to win. Some things can occur by chance or things what you expected may not occur at all. The occurrence or non-occurrence of these things are studied in mathematics by the theory of probability. So in this unit you will learn the simple and introductory concepts of probability.

Figure 6.1

This morning there is a chance of heavy rain with the possibility of thunder. In the afternoon the rain will die away and it is likely that the sun will break through the clouds, probably towards evening.

Weather forecasts are made by studying weather data and using a branch of mathematics called **probability**.

Probability uses numbers to represent how likely or unlikely it is that an event such as 'a thunderstorm' will happen.

Probability is used by governments, scientists, economists, medical researchers and many other people to predict what is likely to happen in the future by studying what has already happened.

Group work 6.1

Discuss with your group.

1. **Find out your classmates' favorite and least favorite school subjects. Ask them what they like and dislike about each.**
2. **Also find out if they like or dislike going to school overall. Find out why or why not.**
3. **Make a chart presenting your findings to the class.**

6.1 The Concept of Probability

Consider a two sided coin. For convenience let the two faces (sides) of the coin be called head (H) and tail (T). If we toss (flip) the coin, the experience shows that the possible outcomes are H (head) or T (tail) but not both. Similarly discuss example 1 with your group.

Example 1

a. Tossing a coin.
b. Tossing two coins.
c. Tossing three coins.
d. Tossing four coins.
e. Rolling a die.

Figure 6.2

Definition 6.1: *A process* of an observation is often called an experiment.

Definition 6.2: The set of all possible out comes of an experiment is called a sample space/the possibility set/ of the experiment and denoted by S.

Definition 6.3: A subset of the sample space of an experiment is called an event and denoted by E.

To illustrate the above definition /explanation/ consider the following summary of probability experiments.

Experiment	Sample space
a. Toss a coin.	{Head, tail} ={H, T}
b. Toss two coins.	{HH, HT, TH, TT}
c. Toss three coins.	{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
d. Roll a die.	{1, 2, 3, 4, 5, 6}
e. Choose an English vowel.	{a, e, i, o, u}
f. Answer a true – false question.	{True, False}

a) Showing head

b) Showing tail

Figure 6.3 coins

Example2: suppose an experiment is rolling a die.

a. List the elements of the sample spaces.
b. List the elements of the set of the event "the number shown is prime".
c. List the elements of the set of the event "the number shown is odd".
d. List the elements of the set of the event "the number shown is even".

Solution

a. The sample space are $\{1, 2, 3, 4, 5, 6\} = S$
b. The prime number is $\{2, 3, 5, \}$ = E
c. The odd number is $\{1, 3, 5, \}$ = E
d. The even number is $\{2, 4, 6, \}$ = E


Figure 6.4 Die

Activity 6.1

1. Identify the following events has certain or impossible events.
- **a. You will grow to be 30 centimeters tall.**
- **b. You will live to be 240 years old.**
- **c. You will die.**
- **d. A newly born baby will be a girl.**

2. Give two examples of events that you think
- **a. are impossible.**
- **b. are certain.**

3. Locate each of the following situations on the probability scale.
- **a. You will have match home work to night.**
- **b. Ababy born today was a girl.**
- **c. The local meteorologist predicts a 40% chance that it will rain tomorrow.**
- **d. If will snow in your town in August.**

Definition 6.4: If the probability of an event is one, then it is called certain event.

Example 3: a. Night will follow day.

b. December following November next year.

c. The next person to come into the room will be right handed.

Definition 6.5: If the probability of an event is 0, then it is called *impossible out comes.*

Example 4: a. When water boils it changes to milk.

b. Two lines intersect at four points.

c. You will grow to be 50 meters tall.

d. It will rain to morrow.

Definition 6.6: All probabilities must have a value greater than or equal to 0 and less than or equal to 1 or $0 \le P(E) \le 1$. This can be shown on a probability scale:

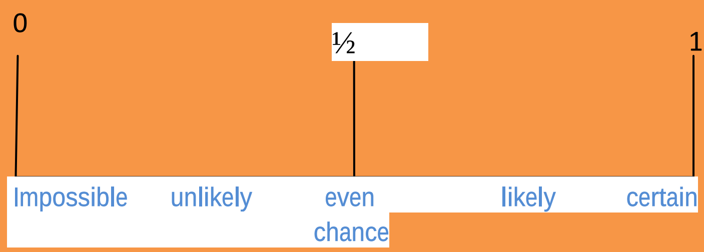


Figure 6.5 Probability scale

Exercise 6A

1. Draw a 0 to 1 probability scale and mark on it the probability that:

   a. the sea will disapper.

   b. you will buy a new pair of shoes soon.

   c. the sun will not rise next week.

d. a member of the class will be late tomorrow.

e. a newely born baby will be a boy.

f. you will watch TV sometime to night.

g. a coin thrown in the air will land heads up.

h. a coin thrown in the air will land tails up.

2. Given two examples of events that you think

a. are impossible

b. are unlikely

c. have about an even chance

d. are likely

e. are certain

6.2 Probability of Simple Events

Group work 6.2

1. A fair six sided die is rolled. What is the probability of getting.

a. a number 4

b. an odd number

c. a multiple of 3

d. less than 5

e. more than 6

f. a number 1 or a number 2

g. an even number

h. 3 or more

i. a prime number

Figure 6.6 die

2. Suppose an experiment is rolling a die:

a. List the elements of the sample spaces.

b. List the elements of the set of the event "the sum of the number is prime.

c. List the elements of the set of the event "the sum of the number is even".

d. List the elements of the set of the event "the sum of the number is 12".

e. List the elements of the set of the event "the sum of the number is 9".

3. Nine playing cards are numbered 2 to 10. A card is selected from them at random. Calculate the probability that the card will be

a. an odd number.
b. a multiple of 4.

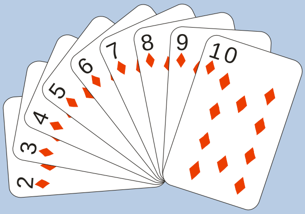


Figure 6.7 Playing Cards

4. What is the probability of an event that is certain occur?
5. What is the probability of an event that cannot occur?

Historical Note

The first book written on the subject of probability was the Book on Games of Chance by Jerome Cardano. He was an Italian physician and mathematician who lived in the 16^{th} century.

Figure 6.8 Jerome Cardan

Definition 6.7: (probability of an event)

In words: The probability of an event is the ratio of the number of successful out comes in the event to the total number of possible out comes in the sample space.

In symbols: P (event) = $\dfrac{\textbf{number of successful outcomes}}{\textbf{total number of possible outcomes}}$

or P(E) = $\dfrac{n(E)}{n(S)}$.

Assuming that the out comes are all equally likely.

Example 5: Find the probability of getting a number 5 or 6 when a fair die is rolled.

Solution:

These are two successful out comes: 5 or 6 and there are six possible out comes: 1, 2, 3, 4, 5, 6 or n(E) = 2 and n (S) = 6.

Thus P(5 or 6) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{2}{6} = \frac{1}{3}$

> **Note:** The word fair means that each number has an equal chance of turning up: the outcomes are equally likely.

Example 6: Find the probability of drawing a card with a prime number on it from a deck of cards numbered l to 20.

Solution:

The sample spaces are $\begin{Bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \end{Bmatrix}$

The events are $\{2, 3, 5, 7, 11, 13, 17, 19\}$.

Hence P(prime number) = $\frac{\text{number of successful out comes}}{\text{total number or possible out comes}} = \frac{8}{20} = \frac{2}{5}$

Example 7: A bag contains 8 discs of which 4 are red, 3 are blue and 1 is yellow. Calculate the probability that when one disc is drawn from the bag it will be

a. red c. blue

b. yellow d. yellow or blue

Figure 6.9

Solution: There are 8 discs altogether so the total number of possible out comes is 8

a. P (red) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{4}{8} = \frac{1}{2}$

b. P(yellow) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{1}{8}$

c. P(blue) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{3}{8}$

d. P(yellow or blue) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{4}{8} = \frac{1}{2}$

Example 8: You randomly draw a slip of paper from a box containing 4 slips. A red slip, a black slip, a white slip and a pink slip.

a. How many possible outcomes are there?

b. What is the probability of each outcomes?

Solution: a. There are 4 possible outcomes.

b. Since each slip is equality likely to be drawn, the probability of each outcomes is $\frac{1}{4}$.

Example 9: A letter is chosen at random from the word PROBABILITY. Work out the probability that it will be:

a. R b. Y c. B d. I or A e. B or I

Solution: Total number of possible outcomes is 11.

a. $P(R) = \dfrac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \dfrac{1}{11}$

b. $P(Y) = \dfrac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \dfrac{1}{11}$

c. $P(B) = \dfrac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \dfrac{2}{11}$

d. $P(I \text{ or } A) = \dfrac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \dfrac{3}{11}$

e. $P(B \text{ or } I) = \dfrac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \dfrac{4}{11}$

Note: An outcome is said to occurre at random if each out comes is equally likely to occur.

Example 10: Six slips of paper one labeled with the letters of "POTATO". The slips are shuffled in a hat and you randomly draw one slip. What is the probability that the slip you draw:

a. the letter T?

b. either the letter O or the letter A?

c. the letter Z?

d. one of the letters P, O. T or A?

Solution: a. $P(t)=\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}=\frac{2}{6}=\frac{1}{3}$

b. $P(O,A)=\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}=\frac{3}{6}=\frac{1}{2}$

c. $P(Z)=\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}=\frac{0}{6}=0$

d. $P(P, O, T \text{ or } A)=\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}=\frac{6}{6}=1$

Example 11: A bowl of fruit contains 3 apples, 4 bananas, 2 lemon and 1 orange. Abebe takes one piece of fruit without looking. What is the probability that he takes.

a. an apple c. a lemon

b. a banana d. an orange

Write each answer in three ways, that means

i. as a fraction

ii. as a decimal and

iii. as a percentage

Figure 6.10

Solution: A bowl of fruit contains = 3 apples + 4 bananas + 2 lemon + 1 orange = 10

a. $P(\text{Lemon})=\frac{\text{number of successful out comes}}{\text{total number of possible out comes}}=\frac{3}{10}$.

Therefore i. probability of an apple as a fraction is $\frac{3}{10}$.

ii. probability of an apple as a decimal is 0.3.

iii. probability of an apple as a percentage is $\frac{3}{10}\times 100\% = 30\%$.

b. $P(\text{banana})=\frac{\text{number of successful out comes}}{\text{total number of possible out comes}}=\frac{4}{10}$.

Therefore, i. probability of an apple as a fraction is $\frac{4}{10}$.

ii. probability of an apple as a decimal is 0.4.

iii. probability of an apple as a percentage is 40%.

c. $P(\text{lemon})=\frac{\text{number of successful out comes}}{\text{total number of possible out comes}}=\frac{2}{10}=\frac{1}{5}$.

Therefore, i. probability of a lemon as a fraction is $\frac{1}{5}$.

ii. probability of a lemon as a decimal is 0.2.

iii. probability of a lemon as percentage is 20%.

d. P(an orange) = $\frac{\text{number of successful out comes}}{\text{total number of possible out ocmes}} = \frac{1}{10}$.

Therefore , i. probability of an orange as a fraction is $\frac{1}{10}$.

ii. probability of an orange as a decimal is 0.1.

iii. probability of an orange as a percentage is 10%.

Example 12: Two dice are rolled. State the probability of each event.

a. The sum is 7.
b. The sum is 13.
c. The sum is less than 13.

Figure 6.11 Dice

Solution: There are 6 numbers on each die. The sample space has 36 or 6^2 out comes.

Sample space

	Second die					
First Die	**1**	**2**	**3**	**4**	**5**	**6**
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

a. The out comes with sum 7 are:
{(1, 6), (2, 5), (3, 4), (4, 3) (5, 2), (6, 1)}.

P(sum =7) = $\frac{\text{number of successful out comes}}{\text{total number of possible out comes}} = \frac{6}{36} = \frac{1}{6}$.

b. There are 0 out comes with sum 13.

$\Rightarrow$ P(sum = 13) = $\frac{0}{36} = 0$.

c. All 36 out comes have a sum less than 13.

P(sum < 13) = $\frac{36}{36} = 1$.

Exercise 6B

1. A counting number less than 30 is chosen at random. What is the probability that the number chosen:
   a. is a multiple of 4?
   b. is a square?
   c. is a cube number?
   d. is a prime?

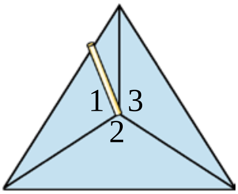
Figure 6.12

2. A jar contains 2 orange, 5 blue, 3 red and 4 yellow marbles. A marble is drawn at random from the jar. Find each probability.
   a. p (orange)
   b. p (red)
   c. p (blue)
   d. p (green)

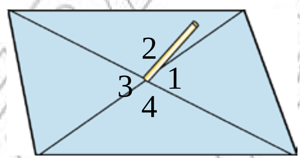
3. Two dice are thrown at the same time. State the probability of each event:
   a. The sum is 5
   b. The sum is 9
   c. The sum is 12

4. A game is played with two spinners. You multiply the two numbers on the spinners land to get the score

Spinner A



Spinner B



This score is 2×4 = 8

Figure 6.13 Spinner

   a. Copy and complete the table to show all the possible scores. One score has been done for you.

Spinner B

Spinner A	×	1	2	3	4
	1				
	2				8
	3				

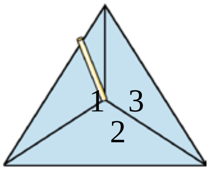
b. Work out the probability of getting a score of 6.

c. Work out the probability of getting a score that is an odd number.

5. This spinner is spun. What is the probability of getting:

a. a number 1

b. an odd number



The three-sided spinner has landed on two

Figure 6.14

6. Nine counters numbered 2 to 10 are put in a bag. One counter is selected at random.

What is the probability of getting a counter with

a. a number 5

b. an odd number

c. a prime number

d. a perfect square number

e. a multiple of 3?

7. A hundred raffle tickets are sold. Raman buys 8 tickets, Susan 5 tickets and Aster 12 tickets. What is the probability that the first prize will be won by

a. one of these three

b. Raman

c. Susan

d. Somebody other than Aster?

Write each answer in three ways:

a. as a fraction

b. as a decimal and

c. as a percentage

Challenge Problem

8. When three dice are thrown at the same time what is the probability that the sum of the number of dots on the top faces will be 6?

9. A number is selected at random from 1 to 100. State the probability that:

a. The number is odd

b. the number is divisible by 5

c. the number is even or divisible by 5

d. the number is divisible by 5 or 3.

Summary for Unit 6

1. *A process of an observation is often called an experiment.*
2. *The set of all possible outcomes of an experiment is called a sample space /the possibility set/ of the experiment and denoted by S.*
3. *A subset of the sample space of an experiment is called an event and denoted by E.*
4. *An event which is certain to happen has a probability of 1.*
5. *An event which cannot happen is (impossible) outcomes has a probability of 0.*
6. *The probability that an event will happen is calculated by:*

   $$\text{Probability of an event} = \frac{\text{number of successful out comes}}{\text{total number of possible out comes}}.$$

7. *The probability of an event happening is always greater than or equal to 0 (impossible) and less than or equal to 1 (certain). This can be written as* $0 \leq \text{probability an event} \leq 1$.

Miscellaneous Exercise 6

I. Write true for the correct statements and false for the incorrect one

1. The probability of an event that is certain to occur is 1.
2. The probability of an event that is an impossible out comes to occur is 0.
3. The probability of getting a sum of 7 or 11 by rolling two dice is $\frac{2}{9}$.
4. If the set of all possible outcomes is equal to an event then the probability of an event is 1.
5. Suppose that two dice are tossed, and then the probability of the sum 1 is also 1.

II. Choose the correct answer from the given alternatives

6. If 2 fair dice are tossed, what is the probability that the sum of the number of dots on the top faces will be 7?

   a. $\frac{1}{9}$ c. $\frac{1}{6}$

   b. $\frac{5}{36}$ d. $\frac{7}{36}$

7. A bag contains 6 white balls, 4 red balls and 5 black balls. If a ball is drawn from the bag at a random, then which of the following is true.
   a. Probability of a white ball is $\frac{2}{5}$.
   b. Probability of red or a black ball is $\frac{3}{5}$.
   c. Probability of not getting a black ball is $\frac{2}{3}$.
   d. All are true

8. A pairs of fair dice is tossed. What is the probability of not getting a sum 5 or 9?
   a. $\frac{2}{9}$ b. $\frac{8}{9}$ c. $\frac{7}{9}$ d. $\frac{1}{9}$
9. Which of the following is true about a probability scale?
   a. Probability of unlikely between 0 and $\frac{1}{2}$.
   b. Probability of even chance is $\frac{1}{2}$.
   c. Probability of likely between $\frac{1}{2}$ and 1.
   d. All are true

III. Work out problems

10. A letter of the English alphabet is chosen at random. Calculate the probability that the letter so chosen be:
    a. vowel
    b. Precedes m and is a vowels
    c. follows m and is a vowels
11. If three coins are thrown. What is the probability of obtaining.
    a. all heads c. at least one heads
    b. all tails d. at least two heads
12. A letter is chosen at random from the words ETHIOPIA MATHS. What is the probability that the letter E is chosen?
13. One letter is chosen at random from the word ISOSCELES. What is the probability of choosing.
    a. the letter C c. a vowel
    b. the letter E d. a consonant