UNIT 3

LINEAR EQUATIONS AND INEQUALITIES

Unit outcomes

After Completing this unit, you Should be able to:

- understand the concept equations and inequalities.
- develop your skills on rearranging and solving linear equations and inequalities.
- apply the rule of transformation of equations and inequalities for solving problems.
- draw a line through the origin whose equation is given.

Introduction

In this unit you will expand the knowledge you already have on solving linear equations and inequalities by employing the very important properties known as the associative property and distributive property of multiplication over addition and apply these to solve problems from real life. More over you will learn how to set up a coordinate plane and drawing straight lines using their equation.

3.1 Further on Solutions of Linear Equations

Group Work 3.1

Discuss with your friends/partners.

1. Solve the following linear equations using equivalent transformation.

   a. $6x - 8 = 26$

   b. $14x + 6x = 64$

   c. $5x - 17 - 2x = 6x - 1 - x$

   d. $5x - 8 = -8 + 3x - x$

2. Solve the following linear equations.

   a. $7(x-1) - x = 3 - 5x + 3(4x - 3)$

   b. $0.60x + 3.6 = 0.40(x + 12)$

   c. $8(x + 2) + 4x + 3 = 5x + 4 + 5(x + 1)$

   d. $8y - (5y - 9) = -160$

   e. $3(x + 7) + (x - 8)6 = 600$

3. Do you recall the four basic transformation rules of linear equations. Explain.

3.1.1 Solution of Linear Equations Involving Brackets

Activity 3.1

Discuss with your friends/partners.

Solve the following linear equations.

a. $4(2x + 3) = 3(x + 8)$

b. $6(5x - 7) = 4(3x + 7)$

c. $4(8y + 3) = 6(7y + 5)$

d. $3(6t + 7) = 5(4t + 7)$

e. $7(9d - 5) = 12(5d - 6)$

f. $10x - (2x + 3) = 21$

To solve an equation containing brackets such as $5(4x + 6) = 50 - (2x + 10)$, you transform it into an equaivalent equation that does not have brackets. To do this it is necessary to remember the following rules.

Note: For rational numbers a, b and c,

a) $a + (b + c) = a + b + c$

b) $a - (b + c) = a - b - c$

c) $a(b + c) = ab + ac$

d) $a(b - c) = ab - ac$

Example 1: Solve: $x - 2(x - 1) = 1 - 4(x + 1)$ Using the above rules.

Solution

$x - 2(x - 1) = 1 - 4(x + 1)$ …Given equation

$x - 2x + 2 = 1 - 4x - 4$ ……. Removing brackets

$x - 2x + 4x = 1 - 4 - 2$………Collecting like terms

$3x = -5$ …………….Simplifying

$\frac{3x}{3} = \frac{-5}{3}$……….…….Dividing both sides by 3

$x = \frac{-5}{3}$……….…….. x is solved.

✓ Check: For $x = \frac{-5}{3}$

$$\frac{-5}{3} - 2\left(\frac{-5}{3} - 1\right) \stackrel{?}{=} 1 - 4\left(\frac{-5}{3} + 1\right)$$

$$\frac{-5}{3} + \frac{10}{3} + 2 \stackrel{?}{=} 1 + \frac{20}{3} - 4$$

$$\frac{-5}{3} + \frac{10}{3} + \frac{6}{3} \stackrel{?}{=} \frac{3}{3} + \frac{20}{3} - \frac{12}{3}$$

$$\frac{11}{3} = \frac{11}{3} \text{ (True)}$$

Example 2: Solve $4(x - 1) + 3(x + 2) = 5(x - 4)$ using the above rules.

Solution

$4(x - 1) + 3(x + 2) = 5(x - 4)$ ……Given equation

$4x - 4 + 3x + 6 = 5x - 20$ ………Removing brackets

$4x + 3x - 5x = -20 + 4 - 6$……...Collecting like terms

$2x = -22$ ……………….Simplifying

$\frac{2x}{2} = \frac{-22}{2}$ ………………..Dividing both sides by 2

$x = -11$ ………..…….X is solved

✓ Check: For $x = -11$

$$4(-11 - 1) + 3(-11 + 2) \stackrel{?}{=} 5(-11 - 4)$$

$$4(-12) + 3(-9) \stackrel{?}{=} 5(-15)$$

$$-48 - 27 \stackrel{?}{=} -75$$

$$-75 = -75 \text{ (True)}$$

Exercise 3A

1. Solve each of the following equations, and check your answer in the original equations.

a. $7x - 2x + 6 = 9x - 32$
b. $21 - 6x = 10 - 4x$
c. $2x - 16 = 16 - 2x$
d. $8 - 4y = 10 - 10y$
e. $8x + 4 = 3x - 4$
f. $2x + 3 = 7x + 9$
g. $5x - 17 = 2x + 4$
h. $4x + 9 = 3x + 17$

2. Solve each of the following equations, and check your answer in the original equations.

a. $7 - (x + 1) = 9 - (2x - 1)$
b. $3y + 70 + 3(y - 1) = 2(2y + 6)$
c. $5(1 - 2x) - 3(4 + 4x) = 0$
d. $3 - 2(2x + 1) = x + 17$
e. $4(8y + 3) = 6(7y + 5)$
f. $8(2k - 6) = 5(3k - 7)$
g. $5(2a + 1) + 3(3a - 4) = 4(3a - 6)$

Challenge problems

3. Solve the equation $8x + 10 - 2x = 12 + 6x - 2$.
4. solve the equation $-16(2x - 8) - (18x - 6) = -12 + 2(6x - 6)$.
5. Solve the equation $(8x - 4)(6x + 4) = (4x + 3)(12x - 1)$.
6. Solve for x in each of the following equations:

a. $m(x + n) = n$
b. $x(a + b) = b(c - x)$
c. $mx = n(m + x)$

3.1.2 Solution of Linear Equations Involving Fractions

Group work 3.2

Discuss with your friends/partners.

1. Work out

a. $\frac{2}{7} + \frac{3}{50}$
b. $\frac{3}{8} + \frac{5}{8} + \frac{7}{8}$
c. $2\frac{9}{10} + 1\frac{5}{8}$
d. $3\frac{2}{5} + 2\frac{7}{15}$
e. $1\frac{3}{4} + 2\frac{5}{16}$

2. **Work out.**
   a. $\frac{21}{4} - \frac{1}{15}$ b. $4\frac{7}{8} - 1\frac{2}{5}$ c. $6\frac{1}{5} - 5\frac{1}{7}$ d. $7\frac{4}{7} - 4\frac{2}{5}$
3. **Work out.**
   a. $\frac{2}{35} \times 2\frac{5}{6}$ b. $2\frac{1}{3} \times \frac{7}{10}$ c. $21\frac{1}{7} \times 1\frac{3}{5}$ d. $3\frac{5}{6} \times 2\frac{5}{7}$
4. **Work out**
   a. $3\frac{5}{9} \div \frac{20}{9}$ b. $36\frac{7}{3} \div 2\frac{2}{5}$ c. $4\frac{3}{5} \div \frac{2}{3}$ d. $2\frac{3}{2} \div \frac{15}{2}$
5. **In a school, $\frac{7}{16}$ of the students are girls. What fraction of the students are boys?**
6. **A box containing tomatoes has a total weight of $5\frac{7}{8}$ kg. The empty box has a weight of $1\frac{1}{4}$kg. what is the weight of the tomatoes?**
7. **A machine takes $5\frac{1}{2}$ minutes to produce a special type of container. How long would the machine take to produce 15 container?**

From grade 5 and 6 mathematics lesson you have learnt about addition, subtraction, multiplication and division of fractions. All of these are shown on the following discussion.

Adding fractions

It is easy to add fractions when the denominators (bottom) are the same:

Easy to add:

$$\frac{35}{29} + \frac{39}{29} = \frac{74}{29}$$

Denominators are the same

what about this?

$$\frac{38}{9} + \frac{37}{11} = ?$$

Denominators are different

Adding fractions with the same denominator

$$\frac{7}{20} + \frac{2}{20} = \frac{9}{20}$$

Add the numerators (top)

Add them over the same denominators, (bottom)

Example 3:

Adding fractions with different denominators

$$\frac{38}{9}+\frac{37}{11}=?$$

First find equivalent fractions to these ones which have the same denominator (bottom):

Fractions equivalent to $\frac{38}{9}$

$$\frac{38\times 11}{9\times 11}=\boxed{\frac{418}{99}}$$

Fractions equivalent to $\frac{37}{11}$

$$\frac{37\times 9}{11\times 9}=\boxed{\frac{333}{99}}$$

$$\frac{418}{99}+\frac{333}{99}=\frac{751}{99}$$

Equivalent to $\frac{38}{9}$

Equivalent to $\frac{37}{11}$

So $\frac{38}{9}+\frac{37}{11}=\frac{751}{99}$

Note: To add fractions, find equivalent fractions that have the same denominator or (bottom).

Subtracting fractions

It is easy to subtract fractions when the denominators (bottom) are the same:

Easy to subtract:

$$\frac{7}{12}-\frac{26}{12}=\frac{-19}{12}$$

Denomintaors are the same

What about this?

$$\frac{5}{9}-\frac{1}{4}=?$$

Denominators are different.

Example 4: (Subtracting fraction with different denominators)

work out $\frac{5}{9}-\frac{1}{4}$

Solution: Find equivalent fractions to these ones which have the same denominator (bottom). An easy way is to change both denominator to 36 because $9 \times 4 = 36$ is LCM of the denominators.

$$\frac{5}{9} \xrightarrow[\times 4]{\times 4} \frac{20}{36} \qquad \frac{1}{4} \xrightarrow[\times 9]{\times 9} \frac{9}{36}$$

$$\frac{20}{36} - \frac{9}{36} = \frac{11}{36}$$

Equivalent to $\frac{5}{9}$ Equivalent to $\frac{1}{4}$

So $\frac{5}{9} - \frac{1}{4} = \frac{11}{36}$

Note: To subtract fractions, find equivalent fractions that have the same denominator (bottom).

Multiplying fractions

To multiply two fractions, multiply the numerators together and multiply the denominators together.

For example,

$$\frac{50}{18} \times \frac{7}{10} = \frac{350}{180}$$

Multiply the numerators (top)

Multiply the denominators (bottom)

You can simplify this to $\frac{35}{18}$ (by dividing the top and bottom of $\frac{350}{180}$ by 10).

Therefore, $\frac{50}{18} \times \frac{7}{10} = \frac{35}{18}$

Dividing fractions

To divide fractions, invert or take the reciprocal of the dividing fraction (turn it upside down) and multiply by the divisor.

For example

Chang the "÷" $\frac{1}{21} \div \frac{3}{7} = ?$

Sign in to a "×" sign

change the fraction you are dividing by up side down. This is called **inverting** the fraction.

$$\frac{1}{21} \times \frac{7}{3} = \frac{1}{9}$$

Now let us consider linear equations having fractional coefficients.

Example 5: Solve $\frac{x+1}{3} + \frac{x-1}{10} = 12$.

Solution: $\frac{x+1}{3} + \frac{x-1}{10} = 12$ …..Given equation

The LCM of the denominators is $3 \times 10 = 30$ since 3 and 10 do not have any common factors.

Therefore, multiplying both sides by 30.

$$30\left(\frac{x+1}{3} + \frac{x-1}{10}\right) = 30\times12$$

$30\left(\frac{x+1}{3}\right) + 30\left(\frac{x-1}{10}\right) = 30\times12$……..By the distributive property

$10(x + 1) + 3(x - 1) = 360$ ………Simplifying

$10x + 10 + 3x – 3 = 360$ ………Removing brackets

$13x + 7 = 360$ ………Collecting like terms

$13x + 7 - 7 = 360 – 7$……Subtracting 7 from both sides

$13x = 353$………Simplifying

$\frac{13x}{13} = \frac{353}{13}$ … … ….Dividing both sides by 13

$$x = \frac{353}{13}$$

The solution set is $\left\{\frac{353}{13}\right\}$.

✓ Check: $\frac{x+1}{3} + \frac{x-1}{10} = 12$

$$\frac{\frac{353}{13}+1}{3} + \frac{\frac{353}{13}-1}{10} \stackrel{?}{=} 12$$

$$\frac{353+13}{39} + \frac{353-13}{130} \stackrel{?}{=} 12$$

$$\frac{366}{39} + \frac{340}{130} \stackrel{?}{=} 12$$

$$\frac{47580+13260}{5070} \stackrel{?}{=} 12$$

$$\frac{60840}{5070} \stackrel{?}{=} 12$$

$$12 = 12 \text{ (True)}$$

Example 6: Solve $\frac{7}{24} = \frac{x}{8} + \frac{1}{6}$.

Solution: The LCM of the denominators is 24.

$24\left(\frac{7}{24}\right) = 24\left(\frac{x}{8} + \frac{1}{6}\right)$ ….Multiplying both sides by 24.

$24\left(\frac{7}{24}\right) = 24\left(\left(\frac{x}{8}\right) + 24\left(\frac{1}{6}\right)\right)$…Distributive property

$7 = 3x + 4$ …………Removing brackets

$7 - 4 = 3x + 4 - 4$………Subtracting 4 from both sides

$3 = 3x$ …………Simplifying

Or $\frac{3x}{3} = \frac{3}{3}$ … … ..Dividing both sides by 3

$x = \frac{3}{3} = 1$

The solution set is {1}.

Example 7: Solve $\frac{x+1}{2} + \frac{x+2}{3} + \frac{x+3}{4} = 16$.

Solution: The LCM of the denominators is 12.

$12\left(\frac{x+1}{2} + \frac{x+2}{3} + \frac{x+3}{4}\right) = (12 \times 16)$…Multiplying both sides by 12.

$12\left(\frac{x+1}{2}\right) + 12\left(\frac{x+2}{3}\right) + 12\left(\frac{x+3}{4}\right) = 12\times16$……Distributive property

$6(x + 1) + 4(x + 2) + 3(x + 3) = 12\times16$……Simplifying

$6x + 6 + 4x + 8 + 3x + 9 = 192$………Removing brackets

$13x + 23 - 23 = 192 - 23$...Subtracting 23 from both sides

$13x = 169$ …………Simplifying

$\frac{13x}{13} = \frac{169}{13}$……….Dividing both sides by 13

$x = 13$

The solution set is {13}.

✓ **Check:** $\frac{13+1}{2} + \frac{13+2}{3} + \frac{13+3}{4} \stackrel{?}{=} 16$

$$\frac{14}{2} + \frac{15}{3} + \frac{16}{4} \stackrel{?}{=} 16$$

$$7 + 5 + 4 \stackrel{?}{=} 16$$

$$16 = 16 \text{ (True)}$$

Example 8: Solve $\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1) = 4$.

Solution: The LCM of the denominators 3 and 2 is 6.

$\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1) = 4$ ………Given equation

$6\left[\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1)\right] = 6\times4$ ……Multiply both sides by 6.

$6\left[\frac{1}{3}(x + 7)\right] - 6\left[\frac{1}{2}(x + 1)\right] = 6\times4$ …Distributive property

$2(x + 7) - 3(x + 1) = 24$ …Simplifying

$2x + 14 - 3x - 3 = 24$ …Removing brackets

$2x - 3x + 14 - 3 = 24$

$-x + 11 = 24$ ….Collecting like terms

$-x + 11 - 11 = 24 - 11$...Subtracting 11 from both sides

$-x = 13$ ……………….Simplifying

$\frac{-x}{-1} = \frac{13}{-1}$………..Dividing both sides by -1.

$x = -13$

The solution set is $\{-13\}$.

✓ Check:

$\frac{1}{3}(x + 7) - \frac{1}{2}(x + 1) \stackrel{?}{=} 4$

$\frac{1}{3}(-13 + 7) - \frac{1}{2}(-13 + 1) \stackrel{?}{=} 4$

$\frac{1}{3}(-6) - \frac{1}{2}(-12) \stackrel{?}{=} 4$

$-2 + 6 \stackrel{?}{=} 4$

$4 = 4$ (True)

Exercise 3B

1. Solve each of the following equations.

a. $\frac{x}{10} = \frac{2}{3}$

b. $\frac{6n}{2} - \frac{3n}{2} = 3\frac{1}{2}$

c. $\frac{-x}{2} + 6 = -3\frac{2}{8}$

d. $\frac{-3}{5} + \frac{x}{10} = \frac{-1}{5} - \frac{x}{5}$

e. $\frac{2x}{5} - \frac{2}{3} = \frac{x}{2} + 6$

f. $\frac{3x}{7} + \frac{35x}{8} = 10$

g. $\frac{5x}{13} + \frac{5x}{26} = 1$

h. $\frac{12}{23} - x = 4$

2. Solve each of the following equations and check your answer in each case by inserting the solution in original equation.

a. $\frac{5x}{6}+\frac{2}{3}=\frac{-1x}{6}-\frac{5}{3}$

b. $\frac{3}{7}x-\frac{1}{4}=\frac{-4x}{7}-\frac{5}{4}$

c. $\frac{24}{5}w+14=62-\frac{6}{10}w$

d. $\frac{9x}{7}-10=\frac{48x}{7}+14$

e. $\frac{2x+2}{2}+\frac{3x+6}{3}+\frac{4x+16}{4}=-6$

f. $\frac{4+2x}{6x}=\frac{12}{5x}+\frac{2}{15}$

g. $\frac{2x+7}{3}-\frac{x-9}{2}=\frac{5}{2}$

h. $\frac{2x+3}{6}-\frac{x-5}{4}=\frac{3}{8}$

Challenge Problems

3. Solve the following equations.

a. $12-\frac{x-2}{2}=\frac{6-x}{4}+\frac{x-4}{4}$

b. $\frac{2x-10}{11}-\frac{2x-4}{7}=10x-17\frac{1}{2}$

c. $\frac{x+9}{4}-\frac{x-12}{5}=6^{2}$

d. $0.78-\frac{1}{25}h=\frac{3}{5}h-0.5$

3.1.3 Solve Word Problems Using Linear Equations

Group work 3.3

1. Two complementary angles are drawn such that one angle is 10° more than seven times the other angle. Find the measure of each angle.
2. A certain number of two digits is three times the sum of its digits, and if 45 be added to it the digits will be reversed; find the number.

Mathematical problems can be expressed in different ways. Common ways of expressing mathematical problems are verbal or words and formulas or open statements. In this sub-unit you will learn how to translate verbal problems to formulas or mathematical expressions so that you can solve it easily. It is important to translate world problems to open statements because it will be clear and concise.

Although there is no one definite procedure which will insure success to translate word problems to open statements to solve it, the following steps will help to develop the skill.

Table 3.1 Problem – solving Flow chart for word problems

Step 1	Read the problem carefully.	• Familiarize yourself with the problem. Identify the unknown, and if possible estimate the answer.
Step 2	Assign lables to unknown quantities	• Identify the unknown quantity or quantities. Let x represent one of the unknowns. Draw a picture and write down relevant formulas.
Step 3	Develop a verbal models	• Write an equation in words.
Step 4	Write a mathematical equation	• Replace the verbal model with a mathematical equation using x or other variable.
Step 5	Solve the equation	• Solve for the variable using steps for solving linear equations.
Step 6	Interpet the results and write the final answer in words	• Once you have obtained a numerical value for the variable, recall what it represents in the context of the problems.
Step 7	Check all answers back into the original statements of the problems	

Example 9: The sum of a number and negative ten is negative fifteen. Find the number.

Solution:

Let x represent the unknown number — Step 1: Read the problem

Step 2: Label the unknown

(a number) + (-10) = -15 — Step 3: Develop a verbal model

x + (-10) = -15 — Step 4: Write the equation

x + (-10) + 10 = -15 +10 — Step 5: Solve for x

x = -5

Therefore, the number is -5. — Step 6: Write the final answer in words.

Example 10: **(Applications involving sales Tax)**

A video game is purchased for a total of Birr 48.15 including sales tax. If the tax rate is 7%. Find the original price of the video game before sales tax is added.

Solution: Step 1: Read the problem

Let x represent the price of the video game.

0.07x represents the amount of sales tax. Step 2: Label variables

$\begin{pmatrix}\text{orignal} \\ \text{price}\end{pmatrix} + \begin{pmatrix}\text{sales} \\ \text{tax}\end{pmatrix} = \begin{pmatrix}\text{total} \\ \text{cost}\end{pmatrix}$ Step 3: Write a verbal equation

x + 0.07x = Birr 48.15 Step 4: Write a mathematical equation

1.07x = 48 .15 Step 5: Solve for x multiply by 100 to clear decimals

100(1.07x) = 100(48.15)

107x = 4815

$\frac{107x}{107} = \frac{4815}{107}$ Step 6: Divide both sides by 107

$x = \frac{4815}{107}$

x = 45 Step 7: Interpret the results and write the answer in words.

Therefore, the original price was Birr 45.

Example 11: **(Applications involving consecutive integers)**

Find three consecutive even numbers which add 792.

Solution: Step 1: Read the problem

Let the smallest even number be x. Step 2: label the unknow

Then the other even numbers are (x+2) and (x+4) Step 3: Develop a verbal model

Because they are consecutive even numbers.

x+(x+2) +(x+4) = 792 Step 4: Write the equation

3x + 6 = 792

3x = 786

x = 262

The three even numbers are 262, 264 and 266. Step 5: Write the final answer in Word

✓ Check

262 + 264 +266 = 792 Step 6: Check

Example 12: **(Applications involving ages)**

The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?

Solution: let m = the man's age and w = the wife's age.

$\Rightarrow$ m + w= 96 ……………Translated equation (1)

$\Rightarrow$ m = 6 + w ……………...Translated equation (2)

$\Rightarrow$ (6 + w) + w = 96 ……... Substituting equation (2) into 1

$\Rightarrow$ 2w + 6 = 96 ……………Collecting like terms

$\Rightarrow$ 2w = 96 – 6 ……………Subtracting 6

$\Rightarrow$ 2w = 90 ………………..Collecting like terms

$\Rightarrow$ w = 45 …………………Divided both sides by 2

Therefore, the age of his wife is 45 years old.

Exercise 3C

Solve each problem by forming an equation.

1. The sum of three consecutive numbers is 276. Find the numbers.
2. The sum of three consecutive odd number is 177. Find the numbers.
3. Find three consecutive even numbers which add 1524.
4. When a number is doubled and then added to 13, the result is 38. Find the number.
5. Two angles of an isosceles triangle are x and (x+10). Find two possible values of x.
6. A man is 32 years older than his son. Ten years ago he was three times as old as his son. Find the present age of each.

Challenge Problems

7. A shop –keeper buys 20kg of sugar at Birr y per kg. He sells 16kgs at Birr $\left(y+\frac{3}{4}\right)$ per kg and the rest at Birr (y + 1) per kg. what is his profit.
8. A grocer buys x kg of potatoes at Birr 1.50 per kg and y kg of onions at Birr 2.25 per kg. how much money does he pay in Birr?

9. If P is the smallest of four consecutive even integers, what is their sum interms of P?
10. The sum of a certain number and a second number is -42. The first number minus the second number is 52. Find the numbers.

3.2 Further on Linear Inequalities

Activity 3.2

Discuss with your friends/partners.

1. Can you recall the definition of linear inequality?
2. Discuss the four rules of transformation of linear inequalities using examples and discuss the result with your teacher.
3. Solve the following linear inequalities.

a. $4x - 16 < 12,\ x \in \mathbb{W}$

b. $\frac{2}{3}x < -4(x-5),\ x \in \mathbb{Z}^+$

c. $20 - \frac{3}{2}x \geq \frac{3}{2}x - 18,\ x \in \mathbb{Q}^+$

d. $0.5(x-8) \leq 10 + \frac{3}{2}x,\ x \in \mathbb{Q}$

From grade 6 and 7 mathematics lesson you have learnt about to solve linear inequalities in one variable based on the given domain.

Example 13: (Solving an inequality)

Solve the inequality $-3x + 8 \leq 22$.

Solution:

$-3x + 8 \leq 22$…………Given inequalities

$-3x + 8 - 8 \leq 22 - 8$ ……..Subtracting 8 from both sides

$-3x \leq 14$ ……Simplifying

$\frac{-3x}{-3} \geq \frac{14}{-3}$…Dividing both sides by -3; reverse the inequality sign

$x \geq \frac{-14}{3}$…….Simplifying

Therefore, the solution set is $\left\{x : x \geq \frac{-14}{3}\right\}$.

Example 14: (Solving an inequality)

Solve the inequality $24x - 3 < 4x + 10,\ x \in \mathbb{Q}$.

Solution:

$24x - 3 < 4x + 10\ x \in \mathbb{Q}$ …………Given inequalities

$24x - 3 + 3 < (4x + 10) + 3$ ……….Adding 3 from both sides

$24x < 4x + 13$…………………Simplifying

$24x - 4x < 4x - 4x + 13$……..Subtracting 4x from both sides

$20x < 13$……………..Simplifying

$\frac{20x}{20} < \frac{13}{20}$……..Dividing both sides by 20

$x < \frac{13}{20}$……….Simplify

Therefore, the solution set is $\left\{x: x < \frac{13}{20}\right\}$.

Example 15: (**Solving an inequality**)

Solve the inequality $4x - 6 > 10$, $x \in \mathbb{N}$.

Solution: $4x - 6 > 10$ …………Given inequalities

$(4x - 6) + 6 > 10 + 6$ ………Adding 6 from both sides

$4x > 16$ ………..Simplifying

$\frac{4x}{4} > \frac{16}{4}$…………..Dividing both sides by 4

$x > 4$

The solution of the inequality is $x > 4$.

Therefore, the solution set is $\{x: x > 4\} = \{5, 6, 7, 8, 9, \dots\}$.

Example 16: (**Solving an inequality**)

Solve the inequality $\frac{-1}{4}x + \frac{1}{6} \leq 2 + \frac{2}{3}x$, $x \in \mathbb{Z}$.

Solution: $\frac{-1}{4}x + \frac{1}{6} \leq 2 + \frac{2}{3}x$………Given inequalities

$12\left(\frac{-1}{4}x + \frac{1}{6}\right) \leq 12\left(2 + \frac{2x}{3}\right)$……Multiply both sides by 12 to clear fractions.

$12\left(\frac{-1}{4}x\right) + 12\left(\frac{1}{6}\right) \leq 12(2) + 12\left(\frac{2}{3}x\right)$…….Apply the distributive property

$-3x + 2 \leq 24 + 8x$ …………Simplifying

$-3x - 8x + 2 \leq 24 + 8x - 8x$ ………Subtracting 8x from both sides

$-11x + 2 \leq 24$ ………..Collecting like terms

$-11x + 2 - 2 \leq 24 - 2$ ………Subtracting 2 from both sides

$-11x \leq 22$ …………..Simplifying

$\frac{-11x}{-11} \geq \frac{22}{-11}$ …………Dividing both sides by -11. Reverse the inequality sign.

$x \geq -2$

There fore, the solution set is {-2, -1, 0, 1, 2, 3, …}.

Example 17: **(Solving an inequality)**

Solve the inequality $3x - 2(2x - 7) \leq 2(3 + x) - 4$, $x \in \mathbb{N}$.

Solution:

$3x - 2(2x - 7) \leq 2(3 + x) - 4$ …Given inequalities

$3x - 4x + 14 \leq 6 + 2x - 4$ ………Removing brackets

$-x + 14 \leq 2x + 2$ ………………...Simplifying

$-x - 2x + 14 \leq 2x - 2x + 2$ ……….Subtracting 2x from both sides

$-3x + 14 \leq 2$ ……………………..Simplifying.

$-3x + 14 - 14 \leq 2 - 14$…………...Subtracting 14 from both sides

$-3x \leq -12$ ……………………….Simplifying

$\frac{-3x}{-3} \geq \frac{-12}{-3}$ …………….Dividing both sides by -3. Reverse the inequality sign

$x \geq 4$

The solution of the inequality is $x \geq 4$.

Therefore, the solution set is $\{x: x \geq 4\} = \{4, 5, 6, 7, 8, 9, \ldots\}$.

Exercise 3D

1. Solve the following inequalities:
   a. $\frac{1}{2}(x + 4) \geq \frac{3}{4}(x - 2)$
   b. $\frac{x}{4} + 5 \leq x + 4$
   c. $8x - 5 > 13 - x$
   d. $4x + 6 > 3x + 3$
   e. $\frac{1}{4}x + 7 \leq \frac{1}{3}x - 2$
   f. $9 + \frac{1}{3}x \geq 4 - \frac{1}{2}x$
   g. $\frac{1}{2}(2x + 3) > 0$
2. Solve each of the following linear inequality in the given domain.
   a. $4 - \frac{5}{6}x > \frac{3}{2}x - 8$, $x \in \mathbb{Q}$
   b. $4y - 6 < \frac{1}{2}(28 - 2y)$, $y \in \mathbb{W}$
   c. $\frac{5}{3}x < -8(x - 6)$, $x \in \mathbb{Z}^+$
   d. $-2(12 - 2x) \geq 3x - 24$, $x \in \mathbb{Q}^+$
   e. $5x + 6 \leq 3x + 20$, $x \in \mathbb{N}$
   f. $\frac{3y}{4} + \frac{1}{6} > \frac{17}{10}$, $y \in \mathbb{Z}$
   g. $6x \geq 16 + 2x - 4$, $x \in \mathbb{Z}$
   h. $10x + 12 \leq 6x + 40$, $x \in \mathbb{N}$
3. Eight times a number increased by 4 times the number is less than 36. What is the number?
4. If five times a whole number increased by 3 is less than 13, then find the solution set.

Challenge Problems

5. Solve each of the following linear inequalities:

   a. $3(x + 2) - (2x - 7) \leq (5x - 1) - 2(x + 6)$

   b. $6 - 8(y + 3) + 5y > 5y - (2y - 5) + 13$

   c. $-2 - \frac{W}{4} \leq \frac{1+w}{3}$

   d. $-0.703 < 0.122x - 2.472$

   e. $3.88 - 1.335t \geq 5.66$

3.3 Cartesian Coordinate System

3.3.1 The Four Quadrants of the Cartesian Coordinate Plane

Group work 3.4

1. Write down the coordinates of all the points marked red in Figure 3.1 to the right .

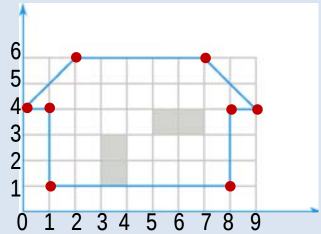

Figure 3.1

2. Write the coordinates of the points A, B, C, D, E, F, G and H shown in Figures 3.2 to the right.

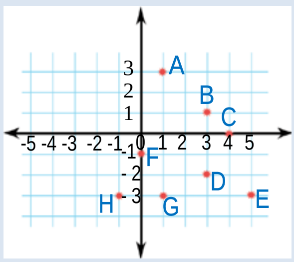

Figure 3.2

3. Name the quadrant in which the point of p(x, y) lines when:

   a. $x > 0, y > 0$

   b. $x < 0, y > 0$

   c. $x > 0, y < 0$

   d. $x < 0, y < 0$

For determing the position of a point on a plane you have to draw two mutually perpendicular number lines. The horizontal line is called the **X –axis**, while the vertical line is called the **Y-axis**. These two axes together set up a plane called the **Cartesian coordinate planes**. The point of intersection of these two axis is called the **origin**. On a suitably chosen scale, points representing numbers on the X-axis are called **X-coordinates or abscissa**, while chose on the y-axis are called **Y-coordinates or ordinat**. The x-coordinate to the right of the y-axis are positive, while those to the left are negative. The y- coordinates above and below the X-axis are positive and negative respectively. Let XOX' and YOY' be the **X-axis and the Y-axis** respectively and let P be any point in the given plane. For determing the coordinates of the point P, you draw lines through P parallel to the coordinate axis, meeting the X-axis in M and the y-axis in N.

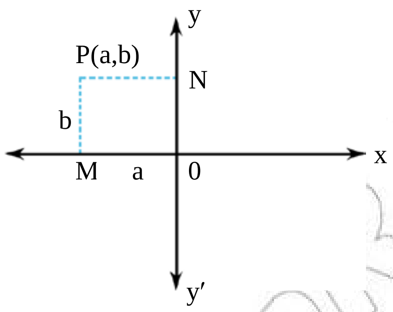

Figure 3.3

The two axes divide the given plane into four quadrants. Starting from the positive direction of the X-axis and moving the anticlockwise (counter clockwise) direction, the quadrants which you come across are called **the first**, **the second**, **the third** and **the fourth quadrants** respectively.

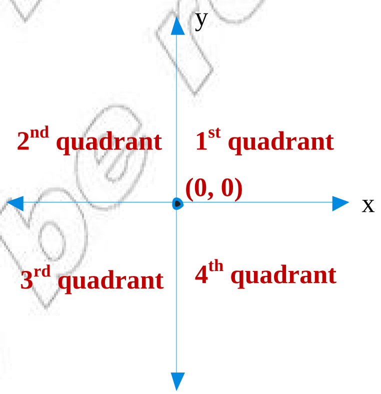

Figure 3.4

Note: i. In the first quadrant all points have a positive abscissa and a positive ordinate.

ii. In the second quadrant all points have a negative abscissa and a positive ordinate.

iii. In the third quadrant all points have a negative abscissa and a negative ordinate.

iv. In the fourth quadrant all points have a positive abscissa and a negative ordinate.

EXERCISE 3E

1. Draw a pair of coordinate axes, and plot the point associated with each of the following ordered pair of numbers.

A(-3, 4)	D (0, -3)	G (0, 6)
B(4, 6)	E (-3, -2)	H (2.5, 3)
C(4, -3)	F (-5, 6)	I (-2, 4.5)

2. Based on the given Figure 3.5 to the right answer the following questions.
   a. Write the coordinates of the point A, B, P, S, N and T.
   b. Which point has the coordinates (-1, -2)?
   c. Which coordinate of the points Q is zero?
   d. Which coordinate of the points D and M is the same?
   e. To which axis is the line DM parallel?
   f. To which axis is the line AT parallel?
   g. If F is any point on the line AT, state its y-coordinate.
   h. To which axis is the line PQ parallel?

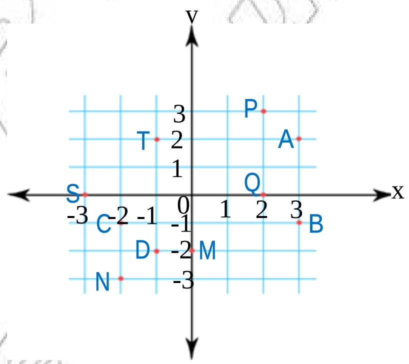

Figure 3.5

Challenge Problems

3. Answer the following:

   a. On which axis does the point A(0,6) lie?

   b. In which quadrant does the point B(-3, -6) lie?

   c. Write the coordinates of the point of intersection of the x-axis and y- axis.

3.3.2 Coordinates and Straight Lines

Group Work 3.5

1. Write down the equations of the lines marked (a) to (d) in the given Figure 3.6 to the right.

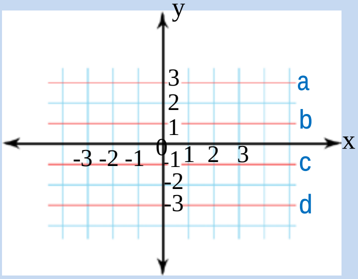

Figure 3.6

2. Write down the equations of the lines marked (a) to (d) in the given Figure 3.7 to the right.
3. Draw the graphs of the following equations on the same coordinate system:

   a. y = x   c. y = 4x

   b. y = -x   d. y = -4x

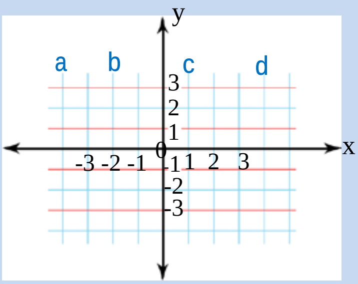

Figure 3.7

4. True or false. If the statement is false, rewrite it to be true.

   a. The line x = 30 is horizontal.   b. The line y = -24 is horizontal.

5. True or false. If the statement is false, rewrite it to be true.

   a. A line parallel to the y – axis is vertical.

   b. A line perpendicular to the x – axis is vertical.

   For exercise 6 – 9, identify the equation as representing a vertical line or a horizontal line.

6. 2x + 7 =10   7. 9 = 3 + 4y   8. -3y + 2 = 9   9. 7 = -2x – 5

10. Write an equation representing the x – axis.

11. write an equation representing the y – axis.

Graph of an equation of the form x = a (a ∈ ℚ)

The graph of the equation x= a (a∈ ℚ, a≠0) is a line parallel to the y-axis and at a distance of **a unit** from it.

Note: i. If a > 0, then the line lies to the right of the y-axis.

ii. If a <0, then the line lies to the left of the y-axis.

iii. The graph of the equation x= 0 is the y-axis.

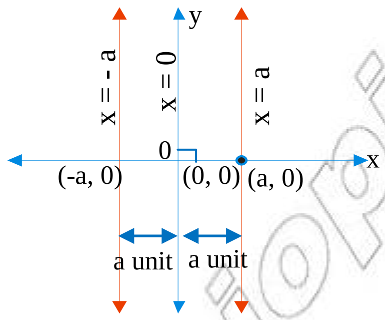

Figure 3.8

Example 18: Draw the graphs of the following straight lines.

a. x = 6   b. x = -6

Solution: First by drawing tables of values for x, and y in which x-is constant and following this you plot these points and realize that the points lie vertical line.

a..

x	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
y	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8

b.

x	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	-5	-4	-3	-2	-1	0	1	2	3	4	5

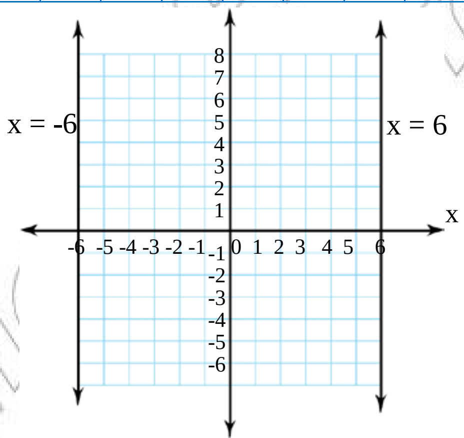

Figure 3.9

Graph of an equation of the form y = b (b ∈ ℚ)

The graph of the equation y= b (b∈ ℚ, b≠0) is the line parallel to the x-axis and at a distance of b from it.

Note: i. If b >0, then the line lies above the x-axis.
ii. If b < 0, then the line lies below the x-axis.
iii. The graph of the equation y= 0 is the x-axis.

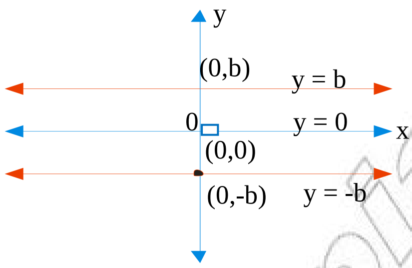

Figure 3.10

Example 19: Draw the graphs of y = 4.

Solution: First by drawing tables of values for x, and y in which y is constant and following this you plot these points and realize that the points lie horizontal line.

x	-4	-3	-2	-1	0	1	2	3	4
y	4	4	4	4	4	4	4	4	4

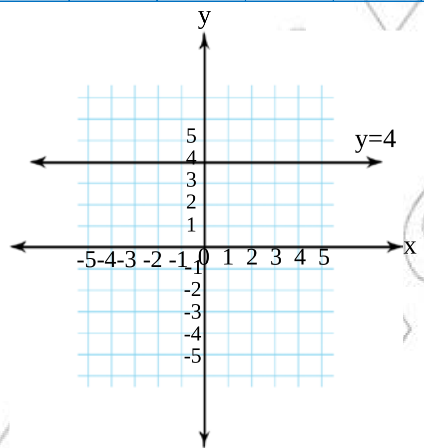

Figure 3.11

Graph of an equation of the form y = mx (m∈ ℚ and m ≠0)

In grade 6 and 7 mathematics lesson we discussed about y = kx, where y is directly proportional to x, with constant of proportionality k. For example y = 4x where y is directly proportional to x with constant of poroportionality 4. Similarly how to draw the graph of y= mx, (m∈ℚ), look at the following examples.

Example 20: Draw the graphs of y = 5x.

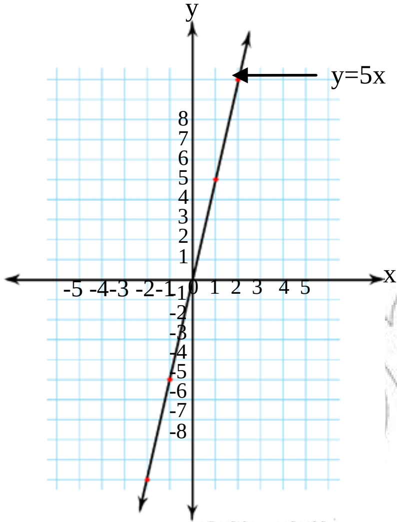

Figure 3.12

Solution:

Step i: Choose some values for x, for example let x= -2,-1, 0, 1 and 2.

Step ii: Put these values of x into the equation y = 5x:

When x= -2: y =5(-2) = -10

When x = -1: y = 5(-1) = -5

When x = 0: y = 5(0) = 0

When x = 1: y = 5(1) = 5

When x = 2: y = 5(2) = 10

Step iii: Write these pairs of values in a table.

x	-2	-1	0	1	2
y	-10	-5	0	5	10

Step iv: Plot the points (-2, -10), (-1, -5), (0, 0) (1, 5) and (2, 10) and join them to get a straight line.

Step v: Lable the line y = 5x.

Example 21: Draw the graphs of y = -5x.

Solution:

Step i: Choose some values for x, for example let x= -2, -1, 0, 1 and 2.

Step ii: Put these values of x into the equation y= -5x

When x= -2: y = -5(-2) =10

When x = -1: y= -5(-1) = 5

When x = 0: y = -5(0)=0

When x= 2: y = -5(2) = -10

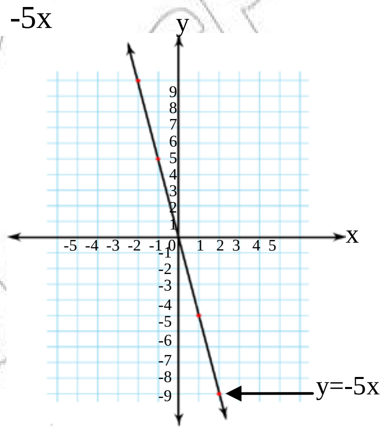

Figure 3.13

Step iii: Write these pairs of values in a table.

x	-2	-1	0	1	2
y	10	5	0	-5	-10

Step iv: Plot the points (-2, 10), (-1, 5), (0, 0) (1, -5) and (2, -10) and join them to get a straight line.

Step v: Label the line y=-5x

3.3.3. The Slope "m" Of Straight Line

Activity 3.3

Discuss with your friends.

1. **What is a slope?**
2. **What is the slope of a line parallel to the y-axis?**
3. **What is the slope of a horizontal line?**
4. **What is the slope of a line parallel to the x – axis?**
5. **What is the slope of a line that rises from left to right?**
6. **What is the slope of a line that falls from left to right?**
7. **a. Draw a line with a negative slope.**
   b. Draw a line with a positive slope.
   c. Draw a line with an undefined slope.
   d. Draw a line with a slope of zero.

From your every day experience, you might be familiar with the idea of **slope**. In this sub – topic you learnt how to calculate the slope of a line by dividing the change in the y – value by change in the x – value, where the y – value is the vertical height gained or lost and the x – value is the horizontal distance travelled.

$$\textbf{Slope} = \frac{\text{change in } y - \text{value}}{\text{change in } x - \text{value}}$$

In Figure 3.14 to the right, consider a line drawn through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$. From P to Q the change in the x coordinate is $(x_2 - x_1)$ and the change in the y coordinate is $(y_2 - y_1)$. By definition, the slope of the line AB is given by:

$$\frac{y_2 - y_1}{x_2 - x_1};\ x_2 \neq x_2$$

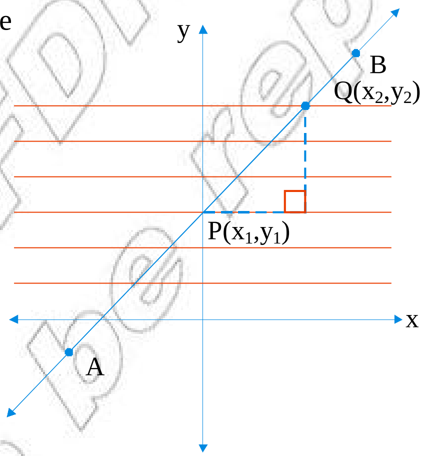

Figure 3.14

Note: If we denote the slope of a line by the letter "m".

Definition 3.1: If $x_1 \neq x_2$ the slope of the line through the points (x_1, y_1) and (x_2, y_2) is the ratio:

$$\text{Slope} = m = \frac{\text{Change in } y - \text{value}}{\text{change in } x - \text{value}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Group work 3.6

Discuss with your friends (partners).

1. In Figure 3.15 below, determine the slope of the roof.

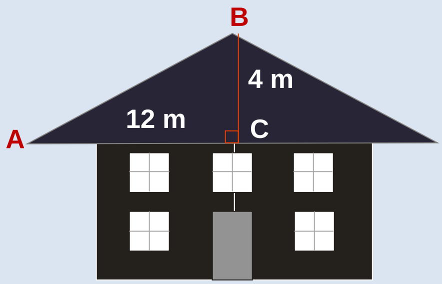

Figure 3.15

2. State the slope of the straight line that contains the points p(1, -1) and Q(8, 10).
3. Find the slope of a line segment through points (-7, 2) and (8,6).
4. Find the slope of each line.

   a) $y = 4$ b) $x = 7$

Example 22: Find the slope of the line passing through the point P(-4, 2) and Q(8, -4).

Solution: $\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{8-(-4)} = \frac{-6}{12} = \frac{-1}{2}$

Therefore, $-\frac{1}{2}$ is the coefficient of x in the line equation $y = -\frac{1}{2}x$.

Example 23: Find the slope of the line passing through each of the following pairs of points.

a) P(4, -6) and Q(10, -6) b) P $\left(\frac{-1}{4}, -4\right)$ and Q$\left(\frac{-1}{4}, 4\right)$

Solution:

a. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(-6)}{10-4} = \frac{-6+6}{6} = \frac{0}{6} = 0$

b. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-4)}{\frac{-1}{4} - \left(\frac{-1}{4}\right)} = \frac{8}{0}$ undefined

Note: **i. The horizontal line has a slope of 0.**
ii. The vertical line has no slope (not defined).

Example 24: Draw the graphs of the following equations on the same Cartesian coordinate plane.

a. $y = \frac{7}{6}x$ b. $y = -3x$ c. $y = 4x$ d. $y = \frac{2}{3}x$

Solution: First to draw the graph of the equation to calculated some ordered pairs that belongs to each equation shown in the table below.

x	-3	-2	-1	0	1	2	3
$y=\frac{7}{6}x$	$\frac{-7}{2}$	$\frac{-7}{3}$	$\frac{-7}{6}$	0	$\frac{7}{6}$	$\frac{7}{3}$	$\frac{7}{2}$
y = -3x	9	6	3	0	-3	-6	-9
y = 4x	-12	-8	-4	0	4	8	12
$y=\frac{2}{3}x$	-2	$\frac{-4}{3}$	$\frac{-2}{3}$	0	$\frac{2}{3}$	$\frac{4}{3}$	2

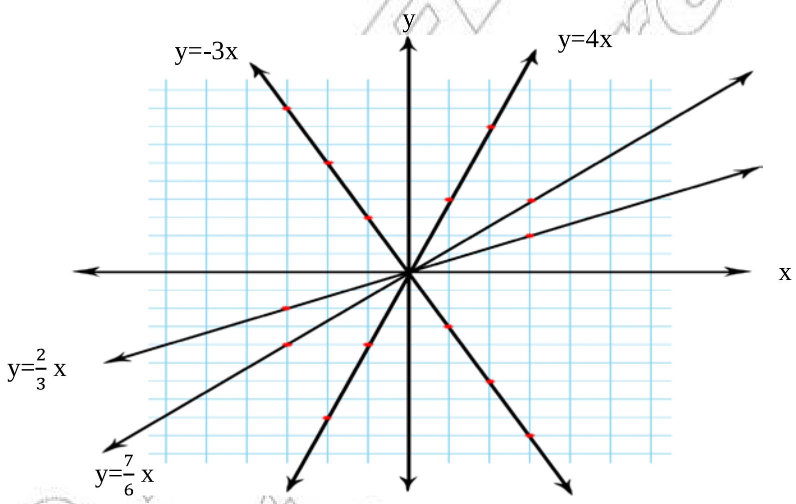

Figure 3.16

From the above graphs, you can generalize that:

i. All orderd pairs, that satisfy each linear equation of the form $y = mx$ $(m \in \mathbb{Q}, m \neq 0)$ lies on a straight lines that pass through the origin.

ii. The equation of the line $y = mx$, m is called the **slope** of the line, and the graph passes the **1st and 3rd quadrants if m > 0**, and the graph passes through the **2nd and 4th quadrants if m< 0.**

EXERCISE 3F

1. Draw the graphs of the following equations on the same coordinate system:

   a. $y= -6x$ b. $y = 6x$ c. $y = \frac{5}{2}x$ d. $y = \frac{-5}{2}x$

2. Draw the graphs of the following equations on the same coordinate system:

   a. $y + 4x = 0$ c. $x = 3$ e. $2x – y = 0$

   b. $2y = 5x$ d. $x + 4 = 0$ f. $\frac{3}{2}x - \frac{y}{2} = 0$

3. Complete the following tables for drawing the graph of $y = \frac{2x}{3}$

x	1	6	3
y			
(x, y)			

Challenge Problems

4. Point (3, 2) lies on the line $ax+2y = 10$. Find a .
5. Point (m, 5) lies on the line given by the equation $5x – y = 20$. Find m.
6. Draw and complete a table of values for the graphs $y=2x -1$ and $y= x- 2$
7. a. Show that the choice of an ordered pair to use as (x_1, y_1) does not affect the slope of the line through (2, 3) and (-3, 5).

   b. Show that $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

   For Exercise 8 – 11 fined the slope of the line that passes through the two points.

8. $P\left(\frac{-2}{7}, \frac{1}{3}\right)$ and $Q\left(\frac{8}{7}, \frac{-5}{6}\right)$
9. $A\left(\frac{1}{2}, \frac{3}{5}\right)$ and $B\left(\frac{1}{4}, \frac{-4}{5}\right)$
10. C (0, 24) and D (30, 0)

11. $E\left(0, \frac{5}{7}\right)$ and $F\left(0, \frac{9}{26}\right)$
12. Find the slope between the points
    A (a + b, 4m – n) and B (a – b, m + 2n)
13. Find the slope between the points
    C(3c – d, s + t) and D (c – 2d, s – t)
14. Write the equation of the line which has the given slope "m" and which passes through the given point.
    a. (2, 10) and m = - 4 b. (4, -4) and m = $\frac{3}{2}$ c. (0, 0) and m = $\frac{3}{5}$
15. State the slope and y-intercept of the line 2x + y + 1 = 0.
16. Find the slope and y-intercept of $y - y_o = m(x - x_o)$ where x_o and y_o are constants.
17. Find the slope and y-intercept of each line:
    a. (x + 2) (x + 3) = (x – 2) (x – 3) + y
    b. x = mu + b
18. State the slope and y-intercept of each linear equations.
    a. 6(x + y) = 3(x – y)
    b. 2(x + y) = 5(y + 1)
    c. 5x + 10y – 20 = 0
19. Write the slope-intercept equation of the line that passes through (2,5) and (-1,3).

Summary For unit 3

1. *You can transform an equation into an equivalent equation that does not have brackets. To do this it is necessary to remember the following rules.*

   a. $a + (b + c) = a + b + c$ **c.** $a\,(b + c) = ab + ac$

   b. $a - (b + c) = a - b - c$ **d.** $a\,(b - c) = ab - ac$

2. *The following rules are used to transform a given equation to an equivalent equation.*

   a. *For all rational numbers a, b and c:*

   If $a = b$ *then* $a + c = b + c$ *and* $a - c = b - c$. *that is, the same number may be added to both sides and the same number maybe subtracted from both sides without affecting the equality.*

   b. *For all rational numbers a, b and c where* $c \neq 0$:

   If $a = b$ *then* $ac = bc$ *and* $\frac{a}{c} = \frac{b}{c}$. *That is both sides may be multiplied by the same non-zero number and both sides may be divided by the same non-zero number without affecting the equality.*

3. *To solve word problems, the following steps will help you to develop the skill. The steps are:*

   a. *Read the problem carefully, and make certain that you understand the meanings of all words.*

   b. *Read the problem a second time to get an overview of the situation being described and to determine the known facts as well as what is to be found.*

   c. *Sketch any figure, diagram or chart (if any) that might be helpful in analyzing the problem.*

   d. *Choose a variable to represent an unknown quantity in the problem.*

   e. *Form an equation containing the variable which translates the conditions of the problem.*

   f. *Solve the equation.*

   g. *Check all answer back into the original statements of the problem.*

4. *The following rules are used to transform a given inequality to an equivalent inequality.*

   a. *For all rational numbers a ,b and c, if* $a < b$ *then* $a + c < b + c$ *or* $a - c < b - c$. *That is, if the same number is added to or subtracted from both sides of an inequality, the direction of the inequality remains unchanged.*

b. For all rational numbers a, b and c

i. If **$a < b$ and $c > 0$,** then **$ac < bc$ or $\frac{a}{c} < \frac{b}{c}$**. That is, if both sides of an inequality are multiplied or divided by the same positive number, the direction of the inequality is unchanged.

ii. If **$a < b$ and $c < 0$,** then **$ac > bc$ or $\frac{a}{c} > \frac{b}{c}$**. That is, if both sides are multipled or divided by the same negative number, the direction of the inequality is **reversed**.

5. The two axes divide the given plane into four quadrants. Starting from the positive direction of the X-axis and moving the anticlockwise direction, the quadrants which you come across are called **the first**, **the second**, **the third** and the **fourth** quadrants respectively.

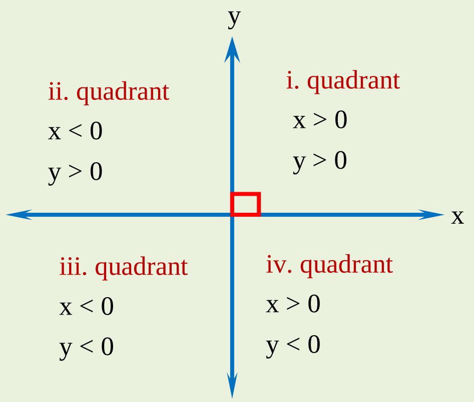

Figure 3.17

6. If $x_1 \neq x_2$ the slope of the line through the points (x_1, y_1) and (x_2, y_2) is the ratio:

$$\textbf{Slope} = m = \frac{\text{Change in } y-\text{value}}{\text{Change in } x-\text{value}}$$

$$= \frac{Y_2 - Y_1}{X_2 - X_1}$$

7. All orderd pairs, that satisfy each linear equation of the form $y = mx$ ($m \in \mathbb{Q}$, $m \neq 0$) lies on a straight lines that pass through the origin.

8. The equation of the line $y = mx$, m is called the slope of the line, and the graph passes the 1st and 3rd quadrants if $m > 0$, and the graph passes through the 2^{nd} and 4^{th} quadrants if $m < 0$.

Miscellaneous Exercise 3

I. Write true for the correct statements and false for the incorrect ones.

1. For any rational numbers a, b and c, then $a(b + c) = ab + ac$.
2. If any rational number $a > 0$, $ax + b > 0$, then the solution set is $\left\{x: x \geq \frac{-b}{a}\right\}$.
3. If any rational number $a < 0$, $ax + b > 0$, then the solution set is $\left\{x: x < \frac{-b}{a}\right\}$.
4. The equation of the line $y = 4x$, 4 is the slope of the line and the graph passes the 2^{nd} and 3^{rd} quadrants, since $4 > 0$.
5. The equation of the line $y= 4x + 6$ that pass through the origin of coordinates.
6. The graphs of the equation $y= b$ ($b \in \mathbb{Q}$, $b \neq 0$), if $b > 0$ then the equation of the line lies above the x-axis.
7. The graph of the equation $x= a$ ($a \in \mathbb{Q}$, $a \neq 0$), if $a < 0$ then the equation of the lines to right of the y-axis.

II. Choose the correct answer from the given alternatives

8. In one of the following linear equations does pass through the origin?

   a. $y = \frac{3}{7}x + 10$ c. $y = \frac{5}{8}x$

   b. $y = -3x + \frac{3}{5}$ d. $y = 2x - 6$

9. The solution set of the equation $\frac{3x+2}{5} - \frac{2x-5}{3} = 2$ is:

   a. $\{1\}$ b. $\{-1\}$ c. $\left\{\frac{1}{2}\right\}$ d. $\left\{\frac{3}{2}\right\}$

10. The solution set of the equation $2x + 3(5 - 3x) = 7(5 - 3)$ is:

    a. $\{5\}$ b. $\frac{1}{7}$ c. $\{3\}$ d. $\left\{\frac{5}{3}\right\}$

11. If $\frac{2}{5x} = 2 + \frac{1}{x}$, $(x \neq 0)$, then which of the following is the correct value of x?

    a. $\frac{1}{8}$ b. $\frac{3}{10}$ c. $\frac{-7}{10}$ d. $\frac{-3}{10}$

12. If x is a natural number, then what is the solution set of the inequality $0.2x - \frac{1}{5} \leq 0.1x$?

    a. $\{x: x \leq 0 \text{ or } x \geq 1\}$ b. ϕ c. $\{1, 2\}$ d. $\{0, 1, 2\}$

13. Which one of the following equations has no solution in the set of integers $\mathbb{Z}$?

    a. $6x + 4 = 10$ c. $9 - 12x = 3$

    b. $8x + 2 = 4x - 6$ d. $\frac{3}{2}x - 3 = 3x$

14. What is the solution set of the inequality $20(4x - 6) \leq 80$ in the set of positive integers?

    a. $\{1, 2, 3, 4\}$ b. $\{1, 2\}$ c. ϕ d. $\{0, 1, 2, 3, 4\}$

15. The sum of the ages of a boy and his sister is 32 years. The boy is 6 years older than his sister. How old is his sister?

    a. 15 b. 19 c. 14 d. 13

III. Work out problems

16. Solve each of the following linear equation by the rules of transformation.

    a. $4x + 36 = 86 - 8x$
    b. $12x - 8 + 2x - 17 = 3x - 4 - 8 + 74$
    c. $4(2x - 10) = 70 + 6x$
    d. $20 - 2x = 62(x - 3)$
    e. $2(6y - 18) - 102 = 78 - 18(y+2)$
    f. $7(x + 26 + 2x) = 5(x + 7)$

17. Sove each of the following equations.

    a. $\frac{2x+7}{3} - \frac{x-9}{2} = \frac{5}{2}$ b. $\frac{x+3}{6} - \frac{x-5}{4} = \frac{3}{8}$ c. $\frac{x+2}{3} + \frac{x+3}{8} = \frac{5}{6}$

18. Solve each of the following linear inequalities by the rules of transformation.

    a. $6x - 2 < 22$ c. $8x - 44 < 12(x - 7)$ e. $\frac{x}{5} - 8 > \frac{-x}{3}$

    b. $-9 \leq 3x + 12$ d. $8(x - 3) \geq 15x - 10$ f. $6(2+6x) \geq 10x - 12$

19. **(word problems)**

    a. The sum of three consecutive odd integers is 129. Find the integers.
    b. Two of the angles in a triangle are complementary. The third angle is twice the measure of one of the complementany angles. What is the measure of each of the angles?
    c. Abebe is 12 years old and his sister Aster is 2 years old. In how many years will Abebe be exactly twice as old as Aster?

20. Draw the graphs of the equations $y = \frac{8}{3}x$ and $y = -\frac{8}{3}x$ on the same coordinate plane. Name their point of intersection as p. State the coordinate of the point p.

21. Find the equation of the line with y-intercept (0,8) and slope $\frac{3}{5}$.

22. Find the slope and y-intercept of $y = 10x - \frac{1}{3}$.

23. Find the slopes of the lines containing these points.

    a) (4,-3) and (6, -4)

    b) $\left(\frac{1}{8}, \frac{1}{4}\right)$ and $\left(\frac{3}{4}, \frac{1}{2}\right)$

    c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ and $\left(\frac{3}{2}, \frac{3}{4}\right)$

24. Find the slope of the line x = -24.

25. Find a and b, if the points P(6,0) and Q (3,2) lie on the graph of ax + by = 12.

26. Points P(3,0) and Q(-3,4) are on the line ax + by = 6. Find the values of a and b.

27. Point (a,a) lies on the graph of the equation 3y = 2x – 4. Find the value of a.

28. Find anequation of the line containing (3,-4) and having slope -2. If this line contains the points (a,8) and (5,b), find a and b.